Definition of De Morgan's Laws:

The complement of the union of two sets is equal to the intersection of their complements and the complement of the intersection of 2 sets is equal to the union of their complements.These are called De Morgan's laws.

For any two finite sets A and B;

(i) $(A \cup B)' = A' \cap B'$ (which is a De Morgans law of union).

(ii) $(A \cap B)' = A' \cup B'$ (which is a De Morgans law of intersection).

Demorgans First Law:

$(A \cup B)' = (A)' \cap (B)'$

The first law states that the complement of the union of two sets is the intersection of the complements.

Proof :

Let $P = (A \cup B)'$ and $Q = A' \cap B'$

Let x be an arbitrary element of P then $x \in P \Rightarrow x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A' \cap B'$

$\Rightarrow x \in Q$

Therefore, $P \subset Q$ .... (i)

Again, let y be an arbitrary element of Q then $y \in Q \Rightarrow y \in A' \cap B'$

Again, let y be an arbitrary element of Q then $y \in Q \Rightarrow y \in A' \cap B'$

$\Rightarrow y \in A'$ and $y \in B'$

$\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)'$

$\Rightarrow y \in P$

Therefore, $Q \subset P$ .... (ii)

Now combine (i) and (ii) we get; $P = Q$ i.e. $(A \cup B)' = A' \cap B'$

Demorgans Second Law

$(A \cap B)' = A' \cup B'$

The second law states that the complement of the intersection of two sets is the union of the complements.

Let $M = (A \cap B)'$ and $N = A' \cup B'$

Let x be an arbitrary element of M then $x \in M \Rightarrow x \in (A \cap B)'$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in A' \cup B'$

$\Rightarrow x \in N$

Therefore, $M \subset N$ .... (i)

Again, let y be an arbitrary element of N then $y \in N \Rightarrow y \in A' \cup B'$

$\Rightarrow y \in A'$ or $y \in B'$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin (A \cap B)$

$\Rightarrow y \in (A \cap B)'$

$\Rightarrow y \in M$

Therefore, $N \subset M$ .... (ii)

Now combine (i) and (ii) we get; M = N i.e. $(A \cap B)' = A' \cup B'$